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Horizon Structure of the AdS_5 Line Element After the Collision of Two Shock Waves

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Abstract In this paper we consider collision of two ultrarelativistic shock-waves in AdS_5 space and discuss about horizon structure of the metric after the collision. We show that all singularities of such metric are hidden behind the horizon.

Keywords *AdS*/CFT correspondence · Black hole · String theory · Brane

1 Introduction

AdS/CFT correspondence [1] is a conjecture deal with string theory or M-theory on the special background of the form $AdS_d \times M_{D-d}$, where AdS_d is an anti de Sitter space of space-time dimension d, and M_{D-d} refers to a certain compact manifold of dimension $D-d$ with $D=10$ for superstring theory and $D=11$ for M-theory. Physically the *AdS* space is solution of the empty space Einstein equation with positive cosmological constant [2]. According to the Maldacena conjecture [1] a supergravity theory such as quantum superstring or M-theory on *AdS* space is mathematically equivalent to the conformally invariant quantum field theory in a $d-1$ dimensional space-time. This ordinary theory lives on boundary of the AdS_d space. After the proposal of Maldacena, other scientists such as Gubser, Klebanov, Polyakov and Witten [3, 4] developed the *AdS*/CFT correspondence by the concept of holography. The famous example of *AdS*/CFT correspondence is the duality between IIB type string theory in $AdS_5 \times S^5$ space and $\mathcal{N}=4$ super Yang-Mills (SYM) gauge theory on the 4-dimensional boundary of AdS_5 space at the large N limit.

Recently, scientists of different areas of physics produced research proposals based on *AdS*/CFT correspondence. Today, one of the interesting field of research is using the *AdS*/CFT to solve complicated problem of QCD in strong coupling phenomena [5–12].

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In the Maldacena dictionary adding temperature to some system in QCD is equivalent to having a black hole (black brane) at the center of *AdS* space. So the Hawking temperature being identified with the temperature in the gauge theory on the boundary of the *AdS* space. Therefore the problem of thermodynamical system in strongly coupled gauge theories becomes related to the problem of the black hole formation [13–15].

Recently, *AdS*/CFT correspondence employed to investigate the collision of two infinite sheets of matter in $\mathcal{N}=4$ SYM in the large N and 't Hooft coupling limit by some researchers [16]. According to the gauge-gravity duality or *AdS*/CFT correspondence, it is equivalent to the collision of two shock waves on the boundary of AdS_5 space. Indeed the shock-wave can be model of the dense medium generated in a heavy ion collision. By using holographic description one can obtain some information about dynamics of the system after collision.

In this article, we shall study such problem by focusing on the horizon structure of the metric after the collision.

2 Collision of Two Shock Waves in AdS_5

In this section, we start our study by a single-shock wave in $\mathcal{N}=4$ SYM with the strong coupling and large N limit. As it is obvious, the AdS_5 line element with a single shock-wave is given by [16, 17],

$$ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4\delta(x^-)dx^{-2} + dx_\perp^2 + dz^2}{z^2}, \tag{1}$$

which is an exact solution of the Einstein equation with $\Lambda = -6$. We work with light-cone coordinates, $x^\pm = \frac{t\pm z}{\sqrt{2}}$. The single shock-wave metric (1) comes from the application of holographic renormalization [18], which relates the metric in Fefferman-Graham form [19] in $d+1$ dimensional space to the energy-momentum tensor in its d dimensional boundary. This is a toy model for a heavy accelerated particle moving nearly at the speed of light along the x^+ or positive $x^3 = z$ direction. In the relation (1), the transverse direction is $x_\perp = (x, y)$, so $dx_\perp^2 = dx^2 + dy^2$. Indeed we take $(x^1, x^2, x^3) = (x, y, z)$. Then as [16] one may obtain the solution after the collision. Also, the line element for after collision is given by the following relation,

$$ds^2 = \frac{-(1+K)d\tilde{\tau}^2 + (1+L)\tilde{\tau}^2 d\tilde{\eta}^2 + (1+H)dx_\perp^2 + (1+M)(1+6\tilde{z}^2\mu\tilde{\tau}\cosh\theta)^2 d\tilde{z}^2)}{\tilde{z}^2(1+2\tilde{z}^2\mu\tilde{\tau}\cosh\theta)^2}. \tag{2}$$

Where $Y - \tilde{\eta} \equiv \theta$, K, L, H and M are arbitrary functions which vanish at initial time $\tilde{\tau} = 0$ [16]. One can determine these functions numerically [20, 21].

Our aim is study of two sheets of matter in a strongly coupled gauge theory in a short time, $\tilde{\tau} \ll 1$ after collision so we restrict ourself to this limit. Therefore just remind second order of $\tilde{\tau}$ and neglect the higher order of $\tilde{\tau}$. Hence one can obtain,

$$\begin{aligned} K(\tilde{\tau},\tilde{z}) &= C\mu^2\tilde{\tau}^2\tilde{z}^4, \\ L(\tilde{\tau},\tilde{z}) &= \frac{C-16}{3}\mu^2\tilde{\tau}^2\tilde{z}^4, \\ H(\tilde{\tau},\tilde{z}) &= -2\mu^2\tilde{\tau}^2\tilde{z}^4, \\ M(\tilde{\tau},\tilde{z}) &= 16\mu^2\tilde{\tau}^2\tilde{z}^4, \end{aligned} \tag{3}$$

where C is the integration constant. It should be noted that, by considering only the second order of $\tilde{\tau}$, the functions (3) is depend only on $\tilde{\tau}$ and $\tilde{z}$ whereas by considering higher order of $\tilde{\tau}$, $\tilde{\eta}$ is also appear in this equation beside $\tilde{\tau}$ and $\tilde{z}$.

Our main goal in this paper is to study of horizon structure of line element presented in (2), but first we would like to have another discussion. If we focus on the minimum value of $\tilde{z}$, where $H(\tilde{\tau},\tilde{z}) = -1$ at early times, we obtain $2\mu^2\tilde{\tau}^2\tilde{z}^4 = 1$ and there is a hidden singularity at $\tilde{z}^4 \leq \frac{1}{2\mu^2\tilde{\tau}^2}$. Therefore we have,

$$\begin{aligned} K &= \frac{C}{2}, \\ L &= \frac{C-16}{6}, \\ M &= 8, \end{aligned} \tag{4}$$

and (2) reduced to the following expression,

$$ds^2 = \frac{-(1+\frac{C}{2})d\tilde{\tau}^2 + \frac{C-10}{6}\tilde{\tau}^2 d\tilde{\eta}^2 + 9(1+3\sqrt{2}\cosh\theta)^2 d\tilde{z}^2}{\tilde{z}^2(1+\sqrt{2})^2}. \tag{5}$$

This equation shows that the horizon is moving towards the boundary in short wile after collision. For one special so called rest-frame central rapidity become $Y = \tilde{\eta}$ and by choosing $C = 10$ we have,

$$ds^2 = \frac{-6d\tilde{\tau}^2 + 9(19+6\sqrt{2})d\tilde{z}^2}{(3+2\sqrt{2})\tilde{z}^2}. \tag{6}$$

In present case there is simple horizon at $z = 0$ just like the original AdS_5 line element with a single shock-wave (1) [16]. Now we are ready to discuss about horizon structure of the black hole solution (2). In addition to the $z = 0$ we obtain other horizon located at the following point,

$$\tilde{z}^2 = -\frac{1}{2\mu\tilde{\tau}\cosh\theta}, \tag{7}$$

but this is not happen because parameters μ, $\tilde{\tau}$ and $\cosh\theta$ are positive and lead to an imaginary $\tilde{z}$. Hence just like previous case we have only one horizon located at the $z = 0$ in the early time.

3 Conclusion

In this paper we used the solution of the collision of two shock-waves in AdS_5 background which discussed already in the [16]. Our main aim was to study of the horizon structure of the line element (2). We learned something about the position of the horizon and found that all singularities of the line element (2) are hidden behind the horizon. We saw that the horizon is moving towards the boundary at small times.

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